

Topics On January Mathematics Final

(Notes By: Michal Aiash/Compilation By: Allen Shalam)

- Laws of Algebra (Packet 1-Chertok)
 - Commutative, Associative, Distributive, Identity Element, Multiplicative Inverse, Math Systems, Evaluating Expressions
- Monomials + Polynomials (Packet 2-Chertok)
 - Add, Subtract, Multiply, Divide
 - Zero, Negative Exponents (Exponential Rules)
 - Scientific Notation
- Equations + Verbal Equations (Packet 3-Chertok)
 - Solving All Equations-Including Parentheses
 - Verbal Problems including
 - Number Problems
 - Consecutive Integers
 - Coin
 - Motion
 - Perimeter-Numerical + Verbal Problems (Equations)
 - Area-Numerical (Shaded Area) + Verbal Problems (Equations)
- Inequalities (Packet 4-Chertok)
 - Graphing On A Number Line
 - Interval Notation- e.g. $()$, $[]$, $(]$, $[)$
 - Verbal Problems → at most, at least, maximum, minimum, etc.
- Ratios and Proportions w/ Verbal Problems (Packet 5-Chertok)
- Circles/3D-Shapes Perimeter, Volume, Surface Area (Packet 6-Chertok)
 - Circumference/Area/Mixed W/ Other Geometric Figures + Shaded Area
 - Surface Area-Rectangular Solid, Cube, Cylinder
 - Volume-All Shapes
- Factoring (Packet 7-Chertok)
- Fractions (Packet 8-Chertok)

- Reducing
- Add
- Subtract
- Divide
- Multiply
- Verbal Quadratic Equations (Packet 9-Chertok)
- Radicals And Square Roots (Packet 10-Chertok)
- Triangles (Packet 11-Chertok)

Mini: Solving Equations

Expression

$$2x+3$$

Equation

$$2x+3=11$$

Balance

$$\begin{array}{r} 2x+3 \\ -3 \quad -3 \\ \hline 2x \quad 8 \end{array} \quad | \quad \boxed{x=4}, \quad \sum x/x=4, \quad \sum x/2x+3=11 = 4$$

$$\begin{array}{r} \frac{3}{5}x - 6 = -18 \\ +6 \quad +6 \\ \hline \frac{3}{5}x - 12 \end{array}$$

$$\rightarrow \frac{4 \times 2}{1} \cdot \frac{5}{3} = 20 \quad \boxed{\text{or}} \quad \frac{3}{5}x - 12 = -18 \quad | \quad +12 \quad | \quad \frac{3}{5}x = -6 \quad | \quad \times \frac{5}{3} \quad | \quad \boxed{x = -10}$$

$$\boxed{x = -20}$$

$$\begin{array}{r} 3x+7 = 5x-3 \\ -3x \quad -3x \\ \hline 7 = 2x-3 \end{array}$$

$$\begin{array}{r} 7 = 2x-3 \\ -2 \quad +3 \\ \hline 10 = 2x \end{array}$$

$$\frac{10}{2} = \frac{2x}{2} \quad \boxed{x = 5}$$

$$\begin{array}{r} 18 \\ 590 \\ \hline 40 \end{array}$$

$$\begin{array}{r} 2x+36 = -3x-54 \\ -2x \quad -2x \\ \hline 36 = -5x-54 \end{array}$$

$$\begin{array}{r} 36 = -5x-54 \\ +54 \quad +54 \\ \hline 90 = -5x \end{array}$$

$$\frac{90}{-5} = \frac{-5x}{-5} \quad | \quad \boxed{x = -18}$$

Michael

39

4. W. p. 122

23. $\frac{1}{4}x - 11 = 5$

$\frac{1}{4}x = 16$ $x = 64$

4. $34 - 0.2$

4.2

24. $13 = 5 - \frac{1}{2}y$

$-\frac{1}{2}y = -8$ $y = 16$

25. $\frac{1}{3}t + 7 = 47$

$-\frac{1}{3}t = -40$ $t = 120$

27. $15x + 14 = 19$

$-14 - 14$

$\frac{15}{15} = \frac{5}{15}$

28. $8 = 18c$

$\frac{1}{18} \cdot 8 = 18c \cdot \frac{1}{18}$

$\frac{8}{18} = c$

$c = \frac{4}{9}$

combine 3 parts with

$t = 1.4$

$16 = x$

$5 - x = 1 - x$

$5 \times 6 = 1$

$\frac{30}{5} = \frac{1}{5}$

$147 = 2x = 2(18 + x)$

$147 = 2x + 36 + 2x$

$111 = 4x$ $\frac{111}{4} = x$

29

14

40

H.W. p. 132

5. $5c = 28 + c$

~~$-c$~~
 $\frac{4c}{4} = \frac{28}{4}$

$= 7$

26. $7y - 5 = 9y + 29$

~~$-7y$~~
 $-5 = 2y + 29$
 ~~-29~~
 $\frac{-34}{2} = \frac{2y}{2}$

$= -17$

10. $8y = 90 + 2y$

~~$+2y$~~
 $\frac{10y}{10} = \frac{90}{10}$

9

19. $4y + 20 = 5y + 9$

~~$-4y$~~
 $20 = 1y + 9$
 ~~-9~~
 $\frac{11}{1} = \frac{1y}{1}$

$= 11$

20. $7x + 8 = 6x + 1$

~~$-7x$~~
 $+8 = -1x + 1$
 ~~-1~~
 $+7 = -1x$
 ~~-1~~

$= -7$

21. $x + 4 = 9x + 4$

~~$-x$~~
 $4 = 8x + 4$
 ~~-4~~

0

22. $9x - 3 = 2x + 46$

~~$-9x$~~
 $-3 = -7x + 46$
 ~~-46~~
 $\frac{-49}{-7} = \frac{-7x}{-7}$

7

Michael

Am: Number Problems

1) Find 1 number
Legend \rightarrow Let # = x

If 3 times a # is increased by 22, the result is 14 less than 7 times the #. Find the #.

$$\begin{array}{r}
 3x + 22 = 7x - 14 \\
 -3x \qquad -3x \\
 \hline
 +22 = 4x - 14 \\
 +14 \qquad +14 \\
 \hline
 36 = 4x \\
 \frac{36}{4} = \frac{4x}{4} \qquad x = 9
 \end{array}$$

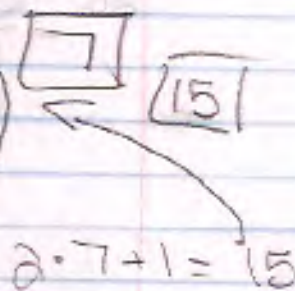
2) Find 2 #s
"exceeding" =

The greater of 2 numbers is one more than twice the smaller. Three times the greater exceeds 5 times the smaller by 10. Find the numbers

Let the smaller # = x
larger # = 2x + 1

$$\begin{array}{r}
 3(2x + 1) = 5x + 10 \\
 6x + 3 = 5x + 10 \\
 -3 \qquad -3 \\
 \hline
 x = 7
 \end{array}$$

$$\begin{array}{r}
 6x = 5x + 7 \\
 -5x \quad -5x \\
 \hline
 x = 7
 \end{array}$$



The second of three #s is one less than the first. The third # is 5 less than the second. If the first # is twice as large as the third, find the three #s.

(42)

$$3x + 22 = 7x - 14$$

$$\begin{array}{r} 3x + 22 \\ -7x - 14 \\ \hline -4x + 36 \end{array}$$

Hold. p. 133

38. 6 times a # = 3 times the number increased by 24. Find the #.

$$6x = 3x + 24$$

$$\begin{array}{r} 6x = 3x + 24 \\ -3x - 3x \\ \hline 3x = 24 \\ \frac{3x}{3} = \frac{24}{3} \end{array}$$

$$x = 8$$

39. If 3 times a # is increased by 22, the result is 14 less than 7 times the #. Find the #.

$$3x + 22 = 7x - 14$$

$$\begin{array}{r} 3x + 22 \\ -7x - 14 \\ \hline -4x + 36 \\ +22 = +4x - 14 \\ +14 \quad +14 \\ \hline 36 = 4x \\ \frac{36}{4} = \frac{4x}{4} \end{array}$$

$$x = 9$$

40. The greater of two #s is 1 more than twice the smaller. 3 times the greater exceeds 5 times the smaller by 10. Find the #s.

smaller - x

$$\text{big} = 2x + 1$$

$$3(2x + 1) = 5x + 10$$

$$x = 7 \quad \text{bigger} = 15$$

41. The second of three #s is 6 more than the first. The third # is twice the first. The sum of the three #s is 26. Find the 3 #s.

$$1 - x = 5$$

$$2 - x + 6 = 11 \quad x + x + 6 + 2x =$$

$$3 - 2x = 10$$

$$4x + 6 = 26$$

$$-6 = -6$$

$$4x = 20$$

$$\frac{4x}{4} = \frac{20}{4}$$

$$x = 5$$

43

57

42. The second of three #s. is 1 less than the first. The third # is 3 less than the second. If the first # is twice as large as the third, find the three #s.

$$\begin{array}{l}
 1 \rightarrow x \rightarrow 12 \quad x = 2(x-6) \\
 2 \rightarrow x-1 \rightarrow 11 \quad x = 2(x-1) \\
 3 \rightarrow x-6 \rightarrow 6 \quad \frac{2x}{2} = 12 \\
 \hline
 -1x \quad -12 \quad - \\
 -1x \quad -12 \quad - \\
 \hline
 \boxed{x=12}
 \end{array}$$

43. It took the Browns family 2 days to travel 925 mi to their home. They traveled 75 mi more on the first day than on the second. How many mi. did they travel each day.

$$\begin{array}{l}
 1 - x + 75 = 500 \quad 2x + 75 = 925 \\
 2 - x = 425 \quad \quad \quad -75 \\
 \hline
 \frac{2x}{2} = \frac{850}{2} \quad \boxed{x=425}
 \end{array}$$

44. During the first 6 months of last year, the interest on an investment was \$130 less than during the second 6 months. The total interest for the year was \$1450. What was the interest for each 6 month period?

$$\begin{array}{l}
 1. 6 months = x - 130 \rightarrow 600 \\
 2. 6 months = x + 130 \\
 \hline
 2x - 130 = 1450 \\
 \quad \quad \quad + 130 \\
 \hline
 2x = 1580 \\
 \quad \quad \quad \div 2 \\
 \hline
 x = 790 \\
 \quad \quad \quad - 130 \\
 \hline
 660
 \end{array}$$

48. Kisha drove from Buffalo to Syracuse at an avg. rate of 48 mph. On the return trip along the same road she was able to travel at an avg. rate of 60 mph. The trip from Buffalo to Syracuse took 1/2 hr. longer than the return trip. How long did the return trip take?

Rate * Time = Distance

going	48	$x + \frac{1}{2}$	$48(x + \frac{1}{2})$
return	60	x	$60x$

$$\begin{array}{l}
 48(x + \frac{1}{2}) = 60x \\
 48x + 24 = 60x \\
 -48x \quad -48x \\
 \hline
 24 = 12x = 2
 \end{array}$$

All Consecutive Integers

- 1, 2, 3, ...
- 2, 3, 24, 25, ...
- 4, -5, -6, ...

Let 1st consecutive integers = x

2nd " " = $x + 1$

3rd " " = $x + 2$

Even consecutive integers

- 2, 4, 6, 8, ...

54, 56, 58, ...

- 2, -4, -6, -8, ...

Let 1st consecutive even integer = x

2nd " " = $x + 2$

3rd " " = $x + 4$

Consecutive odd integers

- 1, 3, 5, 7, 9, ...

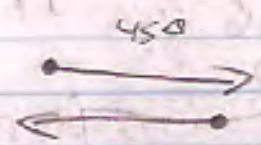
- 1, -3, -5, ...

Let the 1st consecutive odd integer = x

2nd " " = $x + 2$

3rd " " = $x + 4$

4th " " = $x + 6$



before noon	60	$8 - x - 5$	$60(8 - x)$	7 AM
after noon	50	$x - 3$	$50x$	3 PM

$R \times T = D$

$60(8 - x) + 50x = 450$

$480 - 60x + 50x = 450$

$480 - 10x = 450$

$$\begin{array}{r}
 480 - 10x = 450 \\
 -480 \qquad -480 \\
 \hline
 -10x = 30 \qquad +3 \\
 \hline
 x = -3
 \end{array}$$



Sheets

13. $x + x + 1x + 2 = -18$

$$3x + 2 = -18$$

$x = -7$

$x + 1 = -6$

$x + 2 = -5$

14.

$$\frac{3x - 21}{3} = \boxed{-7}$$

15. $x + x + 2 = 86$

$$2x + 2 = 86$$

$$-2 \quad -2$$

$$\frac{2x}{2} = \frac{84}{2} = \boxed{42}$$

$x = 42$

$x + 2 = 44$

20. $x + x + 2 + x + 4 + x + 6 = 450$

$$4x + 12 = 450$$

$$-12 \quad -12$$

$$\frac{4x}{4} = \frac{444}{4} = \boxed{111}$$

$x = 111$

$x + 2 = 113$

$x + 4 = 115$

$x + 6 = 117$

23.

$$x + 4(x + 2) = 3x + 29$$

$x = 21$

$$x + 2 \quad 4x + 8 = 3x + 29$$

$x + 2 = 23$

$$-x \quad -8$$

$$4x = 3x + 21$$

$$-3x \quad -3x$$

$$\boxed{x = 21}$$

24.

$$2x = 3(x + 2) - 26$$

$x = 120$

$$2x = 3x + 6 - 26$$

$x + 2 = 22$

$$2x = 3x - 20$$

$$-3x \quad -3x$$

$$\frac{2x}{-1} = \frac{-20}{-1} = \boxed{+20}$$

26.

$$2x + 1 = x + 26$$

$x = 25$

$$-x \quad -x$$

$x + 1 = 26$

$$x + 1 = 26$$

$x + 2 = 27$

$$-1 \quad -1$$

$$\boxed{x = 25}$$

46

28. $4(x+1) = 3x+23$

$x = 19$

$4x+4 = 3x+23$

$x+1 = 20$

$\frac{4x+4}{-3x} = \frac{3x+23}{-3x}$

$1x+4 = 23$

$-x - 4 = -4$

$\frac{-x}{-x} = \frac{+19}{-1} = \boxed{19}$

29. $x+x+2+x+4 = x+6+18$

$x = 9$

$3x+6 = x+24$

$x+2 = 11$

$\frac{3x+6}{-1x} = \frac{x+24}{-1x}$

$x+4 = 13$

$2x+6 = 24$

$x+6 = 15$

$\frac{2x}{2} = \frac{18}{2} = \boxed{9}$

30. $x+4-3(x+2) = x-47$

$x = 15$

$x+4-3x-6 = x-47$

$x+2 = 17$

$-2x-2 = x-47$

$x+4 = 19$

$+2 + 2 = +2$

$-2x = x-45$

$\frac{-2x}{-1x} = \frac{x-45}{-1x}$

$\frac{-3x}{-3} = \frac{-45}{-3} = \boxed{+15}$

$$\begin{array}{r} 15 \\ 3 \overline{)45} \\ \underline{15} \end{array}$$

33. $x+x+1+x+2 = 3(x+1)$

$x =$

$3x+3 = 3x+3$

$x+1$

$x+2$

(47)

4) Motion Problems

4) objects moving in opposite directions

slow plane	x	5	$5x$
fast plane	$x+60$	5	$5(x+60)$
Rate \times Time = Distance			

$$5x + 5(x+60) = 5300$$

$$5x + 5x + 300 = 5300$$

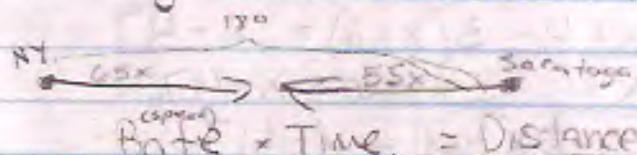
$$10x + 300 = 5300$$

$$-300 \quad -300$$

$$\frac{10x = 5000}{10} = \boxed{500}$$

$x = 500$ mph
$x+60 = 560$ mph

3) objects moving towards each other



NY	65 mph	$x \frac{1}{2}$	$65x$	<div style="border: 1px solid black; padding: 5px; display: inline-block;"> 97.5 miles 82.5 miles </div>
Saratoga	55 mph	$x \frac{1}{2}$	$55x$	

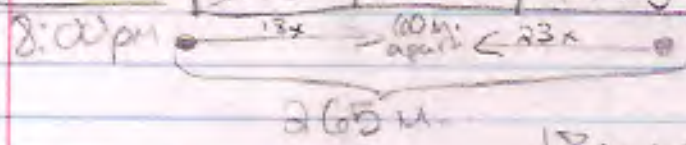
$$65x + 55x = 180$$

$$\frac{120x = 180}{120} = \frac{1}{2}$$

"Gap Problem"

Rate \times Time = Distance

start at 3:00 PM + 5:00 8:00 PM	Ship 1	18 mph	$x-5$	$18x$
	Ship 2	23 mph	$x-6$	$23x$



8:00 PM

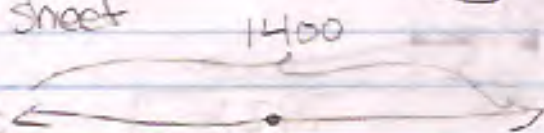
$$18x + 23x = 265 - 60$$

$$\frac{41x = 205}{41} = 5$$

H.W. sheet

(48)

5.



Rate x Time = Dist.

train 1	x	10	10x
train 2	x + 5	10	10(x + 5)

$$\begin{array}{r} 67.5 \\ 20 \overline{) 1350} \\ \underline{130} \\ 50 \\ \underline{40} \\ 100 \end{array}$$

$$10x + 10(x + 5) = 1400$$

$$10x + 10x + 50 = 1400$$

$$20x + 50 = 1400$$

$$x = 67.5$$

$$\begin{array}{r} 20x + 50 = 1400 \\ \underline{20x} \\ 50 \\ \underline{50} \\ 0 \end{array}$$

6.



Rate x Time = Dis.

train 1	x	5	5x
2	x + 20	5	5(x + 20)

$$\rightarrow 55$$

$$\rightarrow 75$$

$$5x + 5(x + 20) = 650$$

$$5x + 5x + 100 = 650$$

$$10x + 100 = 650$$

$$\begin{array}{r} 10x + 100 = 650 \\ \underline{10x} \\ 100 \\ \underline{100} \\ 0 \end{array} = 55$$

7.

Panel 1	x	5	5x	300
2	2x	5	5(2x)	600

$$5x + 5(2x) = 4500$$

$$5x + 10x = 4500$$

$$\begin{array}{r} 15x = 4500 \\ \underline{15} \\ 15 \end{array} = 300$$

8.

Before	60	x	60x	7am
After	50	8-x	50(8-x)	3pm

$$60x + 50(8 - x) = 450$$

$$10x + 400 = 450$$

$$x = 5$$

49
3600

a.

plane 1	792	x	792x
" 2	888	x	888x

Time = Distance

$$792x + 888x = 3360$$

$$\frac{1680x}{1680} = \frac{3360}{1680} = \boxed{2}$$

$\begin{array}{r} 5 \\ 3600 \\ - 840 \\ \hline 3360 \end{array}$

11
792
888

1680
+ 1680

3360

a.

there	32 mph	x	32x
back	4 mph	2.25 - x	4(2.25 - x)

Time = Distance

$$32x = 4(2.25 - x)$$

1400

fast	x + 5	10	10(x + 5)
slow	x	10	10x

R x T = D

$$10(x + 5) + 10x = 1400$$

$$10x + 50 + 10x = 1400$$

$$20x + 50 = 1400$$

$$\begin{array}{r} 20x = 1350 \\ 20 \quad 20 \\ \hline 4500 \end{array} \quad 67.5$$

4500

fast	2x	5	10x
slow	x	5	5x

T x T = D

$$10x + 5x = 4500$$

$$\frac{15x}{15} = \frac{4500}{15} = \boxed{300}$$

50

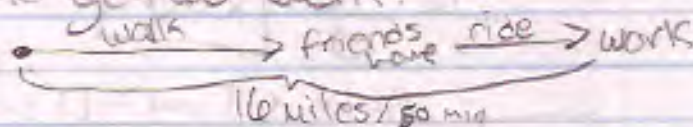
"Round Trip"

Rate x Time = Distance

→	4 mph	X	4X
←	2 mph	6-X	2(6-X)

$$4X = 2(6-X)$$

43. Vera walked from her home to her friend's house at a rate of 3 mph. She rode to work w/ her friend @ a rate of 30 mph. It took Vera 50 min. to walk to her friend's house and to get to work, traveling a total distance of 16 miles. How long did she walk + how long did she ride to get to work.



$$\frac{50}{60} \rightarrow \frac{5}{6} \text{ hour}$$

Rate x Time = Distance

1 st part of trip	3 mph	$X = \frac{1}{3}$	$3X$	20 min.
2 nd part of trip	30 mph	$\frac{5}{6} - X = \frac{1}{2}$	$30(\frac{5}{6} - X)$	$30(\frac{5}{6} - \frac{1}{3}) = 30(\frac{5}{6} - \frac{2}{6}) = 30(\frac{3}{6}) = 15$

$$3x + 30(\frac{5}{6} - x) = 16$$

$$3x + \frac{30}{1} \cdot \frac{5}{6} - 30x = 16$$

$$3x + 25 - 30x = 16$$

$$-27x + 25 = 16$$

$$-25 - 25$$

$$\frac{-27x - 9}{-27 - 27} = \frac{-1}{3}$$

R x T = D

1 st part	3 mph	$50 - x$	$3(50 - x)$	= miles
2 nd part	30 mph	X	30x	

$$3(50 - x) + 30x = 16$$

$$150 - 3x + 30x = 16$$

$$150 + 27x = 16$$

$$-150 \quad -150$$

$$27x = -134$$

51

Type 3: objects go in the same direction

① Difference in time

Sheet #4

Susan 7am. \longrightarrow

Marion 9am. \longrightarrow

Rate \times Time = Distance

Susan	45 mph	$X+2$	$45(X+2)$
Marion	60 mph	$X=6$	$60X$

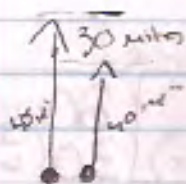
$$45(X+2) = 60X$$

$$45X + 90 = 60X$$

$$\begin{array}{r} -45X \\ \hline \end{array} \quad \begin{array}{r} -45X \\ \hline \end{array}$$

$$\begin{array}{r} 90 \\ 15 \\ \hline \end{array} \quad \begin{array}{r} 15X \\ 15 \\ \hline \end{array} \quad X=6$$

Sheet #11



Rate \times Time = Dist.

slow car	40	$X=6$	$40X$
fast car	45	$X=6$	$45X$

$$45X = 40X + 30$$

$$\begin{array}{r} -40X \\ \hline \end{array} = -40$$

$$\begin{array}{r} 5X \\ 5 \\ \hline \end{array} = \frac{30}{5} = 6$$

OR

$$45X - 40X = 30$$

$$\begin{array}{r} 5X \\ 5 \\ \hline \end{array} = \frac{30}{5} = 6$$

Michael

53

17. 121^m
400 mph \rightarrow

560 mph \rightarrow $R \times T = D$

1 plane	400	X	400X	T
2 plane	560	X+2	560(X+2)	5

2800

$$400x = 560(x+2)$$

$$400x = 560x + 1120$$

$$-560x - 560x$$

$$-160x = -1120$$

$$\frac{-160x}{-160} = \frac{-1120}{-160}$$

$$x = 7$$

19. 32 mph \rightarrow

4 mph \leftarrow $R \times T = D$

rode	32 mph	X	32X	\rightarrow
walks	4 mph	2.25X	4(2.25X)	

$$32x = 4(2.25x)$$

$$32x = 9x$$

$$+ 4x \quad + 4x$$

$$36x = 10$$

$$\frac{36x}{36} = \frac{10}{36}$$

$$x = \frac{10}{36}$$

Muchal

54

H.W. Txbk p. 141 -

42. $\begin{matrix} B \\ \leftarrow \\ \rightarrow \\ A \end{matrix}$

	B	I	D	
1 part	$r-60$	1.5	$1.5r$	99u.
2 "	$r-20$	3	$3(r-20)$	120u.

$$\begin{array}{r} 3 \\ 66 \\ 1.5 \\ \hline 330 \\ 660 \\ \hline 990 \end{array}$$

40

$$1.5r + 3(r-20) = 219$$

$$1.5r + 3r - 78 = 219$$

$$4.5r - 78 = 219$$

$$\begin{array}{r} 66 \\ 4 \overline{) 297} \\ \underline{44} \\ 297 \\ \underline{44} \\ 45 \end{array}$$

66

$$\frac{4.5r}{4.5} = \frac{297}{4.5} \Rightarrow r = 66$$

43. \rightarrow in notes.

57

14

coins * Amount = Total

Quart.	X	25	25x	8
Dimes	20-X	10	10(20-x)	12

$$25x + 10(20-x) = 320$$

$$25x + 200 - 10x = 320$$

$$15x + 200 = 320$$

$$-200 - 200$$

$$\frac{15x}{15} = \frac{120}{15}$$

x = 8

16

bills * Value = Total

1 \$	X	1	1x	10
5 \$	X+1	5	5(X+1)	11
10 \$	X+2	10	10(X+2)	12

$$X + 5(X+1) + 10(X+2) = 185$$

$$X + 5X + 5 + 10X + 20 = 185$$

$$16x + 25 = 185$$

$$-25 - 25$$

$$\frac{16x}{16} = \frac{160}{16} \quad x = 10$$